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FURTHER RESULTS ON K_{e4} DECAY AND ENERGY DEPENDENCE OF LOW-ENERGY-$\pi\pi$-PHASE SHIFT

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From a study of 1609 K_{e4} decays we present: 1) a test of locality in weak interactions; 2) an evaluation of the energy dependence of the $\pi\pi$ phase shift δ_0^0, and of the scattering length a_0.

In two previous publications [1], [2] results of a spark chamber experiment on K_{e4} decay have been presented. The experimental set-up and the data reduction were described there. Results were presented assuming no dependence of the form factors and $\pi\pi$ phase shifts on $M_{\pi\pi}$ and $M_{e\nu}$, the invariant masses of the dipion and dilepton respectively. The range of $M_{\pi\pi}$ extends from 280 to 483 MeV and its mean value in this experiment is 313 MeV.

In this letter we present additional results from the same 1609 events. In the first part, following Pais and Treiman [3], we check the validity of the hypotheses of locality for the leptonic coupling and of the absence of d-wave state for the dipion system. In the second part the $M_{\pi\pi}$ dependence of the form factors and of the $\pi\pi$ phase shifts is presented as well as comparisons with predictions of different $\pi\pi$-scattering models.

Locality. The structure of the intensity distribution in the variable θ_l and φ † is completely determined by the locality of the lepton pair coupling independently of the form of the hadronic current [3]. Projecting the transition probability on the axes of these two variables the following expressions are obtained:

$$dW/d\cos\theta_l = a(1+b\cos\theta_l+c\cos^2\theta_l) \quad (1)$$

$$dW/d\varphi = \alpha(1+\beta\cos\varphi+\gamma\cos 2\varphi+\delta\sin\varphi+\epsilon\sin 2\varphi). \quad (2)$$

These formulae were fitted to the experimental distributions corrected for detection efficiency. Higher order terms were included to search for any improvement in the fits. Results are presented in table 1. Expressions (1) and (2) are in good agreement with the observed distributions and the inclusion of higher terms does not lead to better fits. It should be stressed that the term ϵ in expression (2) is compatible with zero. This coefficient is actually identically zero if the $|\Delta I| = \frac{1}{2}$ rule and T invariance hold. Note that a non-zero δ factor corresponds to the parity violation term in the weak hamiltonian.

d-wave contribution. It has been assumed that only s and p-waves of the $\pi\pi$ system occur in K_{e4} decay. This hypothesis implies [3] the following intensity spectrum in the variable θ_π (decay angle of the dipion) : (3)

$$dW/d(\cos\theta_\pi) = A(1+B\,P_1(\cos\theta_\pi)+C\,P_2(\cos\theta_\pi))$$

where P_1 and P_2 are Legendre polynomials.

A d-wave contribution would require terms in P_3 and P_4. The results of the fit to the experimental distribution are shown in table 1. The coefficients of $P_3(\cos\theta_\pi)$ and $P_4(\cos\theta_\pi)$ are not compatible with zero. However, we believe this result may be spurious. The effect can be explained by the lack of events in one bin in the observed distribution ($0 < \cos\theta_\pi < 0.25$, fig. 3 of ref. [2]‡). Indeed, if this bin is suppressed from

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† For definition, see ref. [3].

‡ The distributions shown in that figure are not corrected for spectrometer acceptance.

Table 1
Expansions of the angular distributions
$dW/d\cos\theta_l \propto 1 + b\cos\theta_l + c\cos^2\theta_l + d\cos^3\theta_l$
$dW/d\varphi \propto 1 + \beta\cos\varphi + \gamma\cos 2\varphi + \delta\sin\varphi + \epsilon\sin 2\varphi + \zeta\cos 3\varphi + \eta\sin 3\varphi$
$dW/d\cos\theta_\pi \propto 1 + B P_1(\cos\theta_\pi) + C P_2(\cos\theta_\pi) + D P_3(\cos\theta_\pi) + E P_4(\cos\theta_\pi)$

b	0.10 ± 0.03	0.10 ± 0.01	β	0.02 ± 0.01	0.02 ± 0.01	B 0.66 ± 0.03	0.67 ± 0.03
c	-0.92 ± 0.03	-0.92 ± 0.01	γ	-0.07 ± 0.02	-0.07 ± 0.01	C 0.21 ± 0.04	0.23 ± 0.04
d		0.00 ± 0.02	δ	-0.13 ± 0.02	-0.13 ± 0.03	D	-0.12 ± 0.04
			ϵ	0.01 ± 0.04	0.01 ± 0.01	E	-0.16 ± 0.08
			ζ		-0.02 ± 0.05		
			η		0.00 ± 0.03		
χ^2/NDF	1.23	1.43		1.01	1.51	1.92	1.05

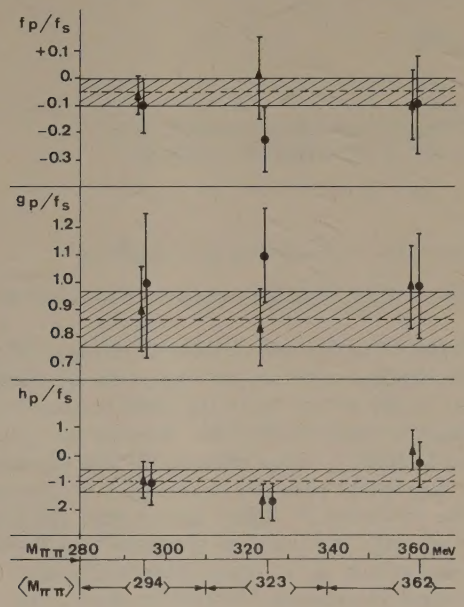

Fig. 1. $M_{\pi\pi}$ dependence of form factors. The dashed area represents the average value with errors obtained on the whole spectrum [2]. The lower line shows the width of the 3 bins and the corresponding mean values. (▲: likelihood points, ●: χ^2 points.)

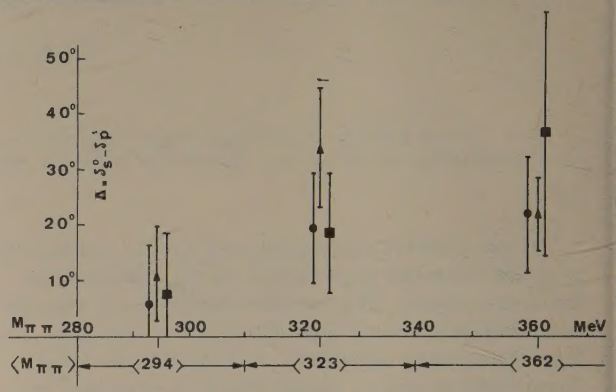

Fig. 2. $M_{\pi\pi}$ dependence of Δ. The meaning of lower line is the same as for fig. 1. (▲: likelihood points, ●: χ^2 points, ■: points from Pais-Treiman method.)

the overall fit the d-wave contribution disappears. Events in this $\cos\theta_\pi$ interval were studied very carefully for possible experimental bias, but they do not exhibit any specific geometrical or kinematical configuration in the laboratory system. Therefore we cannot exclude a d-wave contribution but we think that the observed effect is more likely due to a fluctuation which is not well understood. Also, the suppression of all events with $0 < \cos\theta_\pi < 0.25$ does not change the value of the form factors or the phase shifts.

Form factors and phase shifts. The coefficients in the partial wave expansion of the form factors f_s, f_p, g_p and h_p are real functions of $M_{\pi\pi}$ and $M_{e\nu}$ whereas $\Delta = \delta^0_s - \delta^1_p$ is a function of $M_{\pi\pi}$ only [2]. With the statistics available in this experiment, the full dependence in the two variables cannot be studied. Therefore we restricted ourselves to the $M_{\pi\pi}$ dependence and considered the following three bins:

$280 \leq M_{\pi\pi} \leq 310$ MeV	795 events
$310 < M_{\pi\pi} \leq 340$ MeV	483 events
$340 < M_{\pi\pi}$	331 events.

In each bin the decay parameters were determined by the three methods described in ref. [2] (χ^2 fit on projections, maximum likelihood method and the Pais-Treiman method) assuming all parameters to be constant over the range of each bin. The approximation used in the Pais-Treiman method is explained in the appendix. The results are presented in figs. 1 and 2. The different methods yield consistent values. The slight differences observed can be explained in two ways. First, the physical information is not used in the same way in each method, thus the results are somewhat different for reduced statistics. Second, we may suspect the representation of the errors: although some parameters, mainly f_p and g_p, are strongly correlated, the errors are quoted as if there were no correlations. Such a correlation can be seen in fig. 1: whenever

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Table 2
Fits with formula (4) to the three sets of points of fig. 2

	Maximum likelihood points	χ^2 points	P.T.points
a_0	0.65 ± 0.20	0.55 ± 0.27	0.65 ± 0.33
χ^2/NDF	1.84	0.16	0.34

Table 3
Results of an overall fit with f_p/f_s, g_p/f_s, h_p/f_s and a_0 as parameters

	Maximum likelihood	χ^2
f_p/f_s	-0.05 ± 0.04	-0.09 ± 0.08
g_p/f_s	0.87 ± 0.07	0.93 ± 0.14
h_p/f_s	-0.78 ± 0.37	-0.72 ± 0.36
a_0	0.66 ± 0.17	$0.59^{+0.36}_{-0.27}$

a measurement of f_p is high, the corresponding value of g_p is low.

Fig. 1 does not show any significant $M_{\pi\pi}$ dependence of the form factors within the errors bars. Therefore, we conclude that the assumption of constant form factors is a reasonable one.

Fig. 2 shows the variation of Δ with $M_{\pi\pi}$. (Note that in the analysis Δ was taken as constant over each bin.) Since the p-wave phase shift is small in this energy range [4] we set $\delta_p^1 = 0$ and extract the $I = 0$ $I = 0$ scattering length from our data. We have made no correction for the Coulomb interaction.

$\pi\pi$ scattering length. In order to extract low energy $I = 0$ $\pi\pi$ parameters from our data we parametrize the $\pi\pi$ s-wave phase shift as [4]

$$\text{tg}\,\delta_s^0 = \beta(a_0 m_\pi + 2\alpha(s - 4m_\pi^2)) \qquad (4)$$

where a_0 is the $I = 0$, $I = 0$ scattering length, and where we have

$$s = M_{\pi\pi}^2, \qquad \beta = \sqrt{(s - 4m_\pi^2)/s}\,.$$

In using eq. (4) as a representation of δ_s^0, we assume implicitly that in the energy region of interest, i.e. 280 MeV $\leq M_{\pi\pi} \leq$ 430 MeV, higher order terms in $(s - 4m_\pi^2)$ are unimportant. In particular this implies that no very low energy isoscalar, scalar $\pi\pi$ resonance exists.

The Weinberg current-algebra estimations for the parameters a_0 and α are [5]

$$a_0 \approx 0.2\, m_\pi^{-1},$$
$$\alpha = (32\pi f_\pi^2)^{-1} \approx 2.25 \times 10^{-1}\, m_\pi^{-2},$$

where $f_\pi \sim 95$ MeV is the pion decay constant $(\pi \to \mu\nu)$.

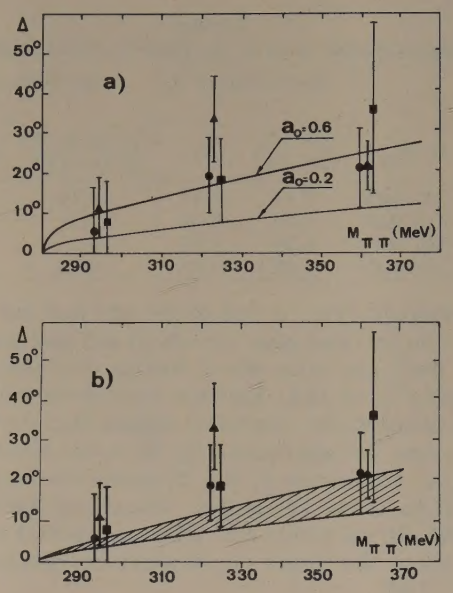

Fig. 3. a) The two curves are obtained by averaging over each bin eq. (4) with two different values of a_0, and α as taken from Weinberg. b) Dashed area is limited by two extreme solutions of the Le Guillou, Morel and Navelet model [6]. Lower curve $a_0 = 0.14\, m_\pi^{-1}$, upper curve $a_0 = 0.23\, m_\pi^{-1}$.

With such values of α, we have determined a_0 by two different methods:

Formula (4) averaged over each bin was fitted to the three sets of points of fig. 2 by varying a_0. Results are presented in table 2. Fig. 3a shows the behaviour of δ_s^0 for $a_0 = 0.6$ and $a_0 = 0.2$ and the experimental points.

In the expression for the transition probability $\sin\delta_s^0$ and $\cos\delta_s^0$ were directly parametrised with formula (4) and an overall fit was made with f_p/f_s, g_p/f_s, h_p/f_s and a_0 as parameters (this approach is not applicable in the Pais-Treiman method). The results are presented in table 3. The values of a_0 are very similar to those of table 2 and the values of the form factors are not significantly changed by the parametrisation of δ_s^0 (compare table 3 of ref. [2] where δ_s^0 was assumed to be constant). In the same manner we investigated whether an energy dependence of f_s or f_p* could alter the value of a_0 but we found no effect.

Conversely, fixing $a_0 = 0.2$, we allowed α to vary. We found that α must then be increased by a factor 5 in order to obtain a reasonable fit. Letting a_0 and α vary simultaneously leads to the same a_0 as before ($a_0 \sim 0.6$) with α poorly

* We used $f_s = f_s' \times \exp(i\delta_s) \times \sin\delta_s/a_0\beta$ and the centrifugal barrier term of ref. [2] for f_p.

Table 4
Approximation used in the Pais-Treiman method

Bins in $M_{\pi\pi}$	Coefficient of a_0		Coefficient of α	
	$\frac{\langle \beta^2 \text{ PS} \rangle}{\langle \beta \text{ PS} \rangle}$	$\langle \beta \rangle$	$\frac{\langle \beta^2 (s-4) \text{ PS} \rangle}{\langle \beta (s-4) \text{ PS} \rangle}$	$\langle \beta (s-4) \rangle$
280 - 493 MeV	0.44	0.43	0.16	0.16
280 - 310 MeV	0.32	0.30	0.47	0.41
310 - 340 MeV	0.50	0.50	0.17	0.17
340 - 493 MeV	0.62	0.63	0.41	0.44

determined. This is due to the fact that our events are concentrated near threshold and that the term $\beta(s-4m_\pi^2)$ has little effect; setting α to zero increases a_0 by 15%. Various theoretical models give phase shifts somewhat higher than the Weinberg one, for approximately the same scattering length. For instance, fig. 3b shows the range of solutions of the Le Guillou, Morel and Navelet model [6]; we notice that all experimental points are above the higher curve which corresponds to $a_0 = 0.23$.

To conclude, the results of this experiment are in agreement with the assumptions of locality, $\Delta I = \frac{1}{2}$ and T invariance. The d-wave test is inconclusive. Since the determination of the s-wave scattering length is somewhat model dependent, a more complete knowledge of the energy dependence of the phase shift may be required before a firm conclusion can be reached. With the present statistics the phase shift of fig. 2 does not rule out $a_0 \sim 0.2$, however, we notice that the effective range predicted by current algebra does not explain its large size, and that the optimal values in our fits are consistently close to $a_0 = 0.6 \pm 0.25$. In the energy range of our experiment, a stronger energy dependence of δ_S^0 than eq. (4) would be required in order to obtain agreement of our data with $a_0 = 0.2$ (Weinberg's value).

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Appendix. Pais and Treiman give the following expression for the phase-shift difference Δ:

$$\tan \Delta(M_{\pi\pi}) = \tfrac{1}{2} I_7'(M_{\pi\pi})/I_4'(M_{\pi\pi}) ,$$

where I_4' and I_7' are two coefficients of the $\mathrm{d}^2 W/\mathrm{d}\cos\theta_l \,\mathrm{d}\phi$ distribution.

In this experiment we must consider average values over a certain range in $M_{\pi\pi}$. The correct expression would be

$$\langle \tan \Delta \rangle = \tfrac{1}{2} \langle I_7'/I_4' \rangle .$$

Unfortunately, the ratio I_7'/I_4' cannot be computed for each value of $M_{\pi\pi}$. Therefore we used the approximation

$$\langle \tan \Delta \rangle = \tfrac{1}{2} \langle I_7' \rangle / \langle I_4' \rangle .$$

One can question the validity of this approximation in the vicinity of the threshold, where Δ is rapidly increasing with $M_{\pi\pi}$. We shall justify it using formula (4). We have

$$I_4' \sim f_S g_P \beta \cos\Delta \text{ PS}$$
$$I_7' \sim 2 f_S g_P \beta \sin\Delta \text{ PS} ,$$

where β is the pion velocity and PS a phase space factor.

Using the crude approximation

$$\cos\Delta = 1, \quad \sin\Delta = \tan\Delta = \beta(a_0 + 2\alpha(s-4))$$

and assuming the form factors to be constant we obtain

$$I_4' \sim \beta \text{ PS}$$
$$I_7' \sim 2\beta^2 (a_0 + 2\alpha(s-4)) \text{ PS} .$$

Then

$$\tfrac{1}{2} \frac{\langle I_7' \rangle}{\langle I_4' \rangle} = a_0 \frac{\langle \beta^2 \text{ PS} \rangle}{\langle \beta \text{ PS} \rangle} + 2\alpha \frac{\langle \beta^2 (s-4) \text{ PS} \rangle}{\langle \beta \text{ PS} \rangle} .$$

This expression has to be compared with

$$\tfrac{1}{2} \langle I_7'/I_4' \rangle = a_0 \langle \beta \rangle + 2\alpha \langle \beta(s-4) \rangle .$$

The coefficients of a_0 and α for the two formulae are listed in table 4 for different bins in $M_{\pi\pi}$; the agreement is seen to be good.

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